EVELIEN P. H. ZEGGELINK

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SOCIAL NETWORK EVOLUTION AND ACTOR ORIENTED MODELS

Applications in the fields of friendship formation, decision making, emergence of cooperation, and coalition formation

Evelien P.H. ZEGGELINK

SUMMARY — We present an overview of different actor oriented models of network evolution, that have been developed in the last couple of years. The models are constructed in different fields of application and all have in common that the emergence of network structure is directly or indirectly of interest. Each model is based on a set of actors and a set of behavioral rules of these actors, resulting in interaction mechanisms and the coming into existence of some network pattern of relationships. Actors vary from individuals and families to political parties. Relationships are either directed or undirected and vary from friendship to cooperation, and access to coalition partners. Simulation is used to obtain distribution of possible resulting network structure because this and other aspects of the models, make it hard to be solved analytically. We think that the use of this kind of simulation models, by examining the influence of both endogeneous and exogeneous variables, contributes to improvement of theory building.

1. ACTOR ORIENTED MODELS OF EVOLUTION OF SOCIAL NETWORKS

To an increasing extent, social networks are seen as being dynamic and dependent. They are no longer examined as static entities that function as independent variables, but their emergence and existence is explained as the result of complex underlying dynamic processes (Stokman and Doreian 1996). Network evolution is not only interesting in its own right but also provides

1 Department of Statistics and Measurement Theory / ICS, University of Groningen, Grote Kruisstraat 2/1, Groningen, The Netherlands, e.p.h.zeggelink@ppsw.rug.nl.
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openings for explanations in the opposite direction: from the network structure to individual behavior. As such, many other studies might benefit from the evolution explanations because social networks have been important for the explanation and description of a variety of social, political, and economic phenomena (Nohria and Eccles 1992, Wasserman and Faust 1994, Wasserman and Galaskiewicz 1994, Weesie and Flap 1990, Wellman and Berkowitz 1988).

In this paper we deal with actor oriented models of network evolution. They have a strong theoretical foundation, but become more and more methodologically appealing (Snijders 1996). The models are based on the striking similarities between the principles of methodological individualism and the principles of object oriented modeling. Following methodological individualism, the network, defined as the macro level, is considered to be the result of interaction mechanisms between the units (individuals or larger social entities) at the micro level (Boudon and Bourricaud 1982, Coleman 1990, Lindenberg 1985). Object oriented modeling or programming is based on the principle that objects act, and react, based on communication between them. Every object has its own characteristic state and characteristic behavior. As such object oriented modeling is very close to theory development, especially in the scope of methodological individualism (Hummon and Fararo 1994). Other advantages are best formulated in comparison with earlier models of network dynamics. Overviews of these models can be found in Snijders (1996), Wasserman and Faust (1994), Zeggelink (1994), and Zeggelink et al. (1996a). Earlier deterministic models had a strong theoretical foundation but were difficult to formalize and test empirically. Most stochastic models were based on well developed statistical techniques but lacked a theoretical basis. Effects from dyadic and triadic levels in the structure were considered as causes for change and dyads and triads were assumed to be conditionally independent. Consequently, a sociological meaningful theory was difficult to incorporate. More important is that dyads and triads are not the best units of analysis because decisions on establishing or dissolving relationships are not made at these levels. It is the actor who takes these decisions and thereby influences dyad, triad and network structure. The implementation of these theoretical notions is facilitated by object oriented models. Restrictions and opportunities that emerge from the macro level and simultaneity of individual actions are implicit in such models. Moreover, heterogeneity of individual actors can easily be incorporated. The models soon become too complex to handle analytically, but simulation is an easily available tool in an object oriented environment (Zeggelink et al. 1996a).

Having described the advantages of an object oriented model for a theoretically meaningful model of network evolution, we first sketch an outline of such an actor oriented model. The model starts with a set of rational actors, each with his own characteristics and behavior. Network emergence is considered to be the effect, possibly unintended, of goal directed behavior of the actors. Every actor has a goal function (utility / tension) which he tries to optimize (utility maximization / tension minimization). Attempts to optimization take place by establishing, maintaining, and breaking up relationships. This behavior is formulated within a set of alternative actions from which the actor may choose. The alternatives and their evaluations depend on objective restrictions and on subjectively perceived restrictions. The alternative is chosen, that according to the expectations of the actor will lead to the largest increase of utility. The real utility increase not only depends on his own actions but also on the (unknown, possibly simultaneous) actions of other actors. Consequently, rationally chosen alternatives do not necessarily lead to the expected results because the actor did not or did not properly take into account the possible behavior of the others. In fact, the actor cannot always reckon with the others' actions because an actor has limited information on the situation and intentions of other actors. However, actors learn. Actors evaluate the ultimate success of their

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2 Whenever we refer to an actor by 'he', we also mean to refer to the female form.
3 An actor may for instance apply so-called myopic behavioral rules, meaning that he can imagine only what would happen at the next point in time as a result of his own actions.
choices and assumptions and adapt their behavior correspondingly. The subsequent step in the model is the aggregation and interaction of behaviors to network structure, followed by feedback to the actors, who on their turn show certain behavior, and so forth.

A number of such dynamic simulation models has been constructed in different fields of application. The models are used to find, by simulation, relationships between independent variables (input parameters) and dependent variables (output parameters) of the network, relevant to the research topic at hand. At this point, we want to deal with the different applications simultaneously, to discover their similarities and differences and in this way obtain ideas for future research. We explain every single model in terms of the following aspects: (1) Research problem and type of network evolution; (2) Type of actors and relationships; (3) Goals of actors; (4) Characteristics of actors and relationships; (5) Information available to the actors; (6) Behavioral rules; (7) Learning principles; (8) Type of dynamics in the model; (9) Independent and dependent variables; (10) Results and empirical tests.

In Section 2, we explain models of evolution of friendship networks (Zeggelink 1993, 1994, 1995, 1997; Zeggelink et al. 1996b). In Section 3 we describe applications in policy networks with respect to decision making processes (Stokman and Van Oosten 1994, Stokman and Zeggelink 1996a; Snijders et al. 1996a, 1996b). The principles of the former two models are combined in Section 4 where we model the simultaneous process of friendship formation and social influence (Stokman and Zeggelink 1996b). Section 5 deals with the evolution of cooperation in primitive populations (De Vos and Zeggelink 1994, 1996). Modeling the formation of coalitions in multi-party democracies is described in Section 6 (Van Roozendaal and Zeggelink 1994, 1996). In Section 7, we summarize the models and present suggestions for future research activities.

2. EMERGENCE OF FRIENDSHIP NETWORKS

2.1. Research topic

Within a population of initially mutual strangers who will interact with each other for a certain time in a specific context, for example children in a classroom, a friendship network will emerge. Friendships emerge as a consequence of each individual's need for social contact and the subsequent spontaneously created joint activities and information exchanges. The network generally is not a random network but shows recognizable patterns based on relevant aspects of friendship formation between individuals. This is especially interesting because friendships are highly subject to individual choice and seem easily changeable. Our main goal is the prediction of global friendship network structure in a population of initially mutual strangers. To achieve this goal, models 1 to 4 have been developed. Here, we focus on the most complex model 4 (Zeggelink 1997) that has been developed from the simpler models 1 to 3.

2.2. Actors and relationships

We consider a closed set of \( g \) individuals who initially are mutual strangers \( A = \{a_i | i = 1, ..., g\} \). No actor can leave the population, and no actor enters the population. The relationship of interest is the undirected friendship relationship without any differences in strength. It is defined as a mutual positive choice. As such, positive choices are the main type of 'ties' and represent the willingness to establish a friendship with the actor to whom the choice is directed. An actor can also send negative messages. These messages are used for modeling convenience
and do not have any direct "negative" meaning: they represent that the sending actor does not want to be friends with an actor that chooses him.\footnote{This does not necessarily mean that he has a negative attitude towards that individual, but represents simply that he is not open to form another friendship.}

2.3. Goals

Dependent on the complexity of the model, goals of actors differ. In model 4 it is assumed that actors aim at establishing a desired number of friendships, want to be friends with actors similar to them, and want to belong to subgroups. Models 1 to 3 consider simpler combinations of these three subgoals.

2.4. Characteristics

All actor characteristics are constant. We assume that every actor has a certain need for social contact, represented by his desired number of friends ($df_i$). Furthermore, every actor has a dichotomous characteristic $x_i$. This characteristic captures all relevant attributes of the actor.

2.5. Information

An actor knows the value of the dichotomous characteristic of all other actors. For models where the group goal is relevant, we assume that an actor is able to observe whether he is a group member or not. If so, other group members and their friends are known.

2.6. Behavioral rules

The behavioral rules are based on tension minimization with respect to so-called issues. An issue is any kind of dimension with respect to friendships one has an opinion about, and one thinks is changeable by one's own actions (Hoede, 1990). It represents one of the subgoals introduced earlier, i.e. the number of friends, characteristics of friends, or state of group membership. An actor's state with regard to the presence and configuration of friendships is evaluated in his tension, and every actor always tries to reduce his tension with respect to the issues.

Let $z$ be the number of issues, and let $\Delta_i(t) \geq 0$ be $a_i$'s tension with respect to the $p^{th}$ issue at time $t$, and let $w_{ip} \geq 0$ be the importance of the $p^{th}$ issue to $a_i$. Then the general form of the tension function for $a_i$ at time $t$ is given by:

$$\Delta_i(t) = \sum_{p=1}^{z} w_{ip} \Delta_{ip}(t)$$

Usually tension $\Delta_{ip}$ is given by some function of the difference between the evaluation of an ideal (preferred) state and the evaluation of the current state, according to $a_i$, on issue $p$.

Closely related to negative messages are waiting periods. Actors do not wait infinitely long for a reciprocated positive choice or a negative message from someone they want to be friends with, i.e. asymmetric positive choices tend to be withdrawn. All actors have a so-called 'waiting period': the maximum amount of time that they will not withdraw an unreciprocated choice. To avoid the problem of interpersonal comparisons of waiting periods, we introduce a so-called 'waiting equilibrium'. This is the situation (configuration of states) that all actors wait for reactions of other actors in the form of reciprocated friendship choices or negative messages, but no actor does make such a choice or send such a negative message because he has no impetus to do so. We assume that the minimal waiting period of all actors is larger than the time the process needs to reach this waiting equilibrium. To keep the process running, one randomly chosen actor (the most impatient), will withdraw (randomly one of) his unreciprocated choice(s).
Model 4 is based on three issues. The first issue represents every actor's specific need for social contact \((df)\). Tension is smaller if the difference between the actual and desired number of friends is smaller:

\[
\Delta_{i1}(t) = |df_i - f_i(t)|,
\]

where \(f_i(t)\) is \(a_i\)'s actual number of friends at time \(t\).

Since another relevant aspect of friendship formation is similarity, the second issue is characteristics of (potential) friends. Since only one dichotomous variable captures all attributes of the actor, an actor can simply distinguish similar and dissimilar actors. \(w_{i2}\) is the importance actor \(a_i\) attaches to similarity:

\[
\Delta_{i2}(t) = \left( df_i - \sum_{j=1}^{f_i(t)} (1 - w_{i2} \sum_{j=1}^{f_i(t)} |x_i - x_j|) \right).
\]

The third issue is the group. In real friendship formations, individuals cannot observe when a group can readily be formed. It is only as a side effect of having friends in common that groups get a chance to develop. Since the definition of the precise functional shape of the group tension function is not that relevant here, we simply put that \(L_i(t)\) is the group to which actor \(a_i\) belongs at time \(t\) (Zeggelink et al. 1996b, Zeggelink 1997). A group is defined as an LS set (Borgatti et al. 1990). If \(a_i\) belongs to no group, \(L_i(t)\) is defined as the set consisting of only \(a_i\) himself. We assume that once \(a_i\) is a group member, he tries to guarantee the future of that group but also takes care of his non-group goals (number and characteristics of friends). If \(a_i\) is a group member, his group sense is stronger (and tension smaller) the smaller his and the group's total number of external friendships (\(\alpha\)), and the larger his and the group's total number of internal friendships (\(\lambda\)). Total tension is larger for non-group members than for group members:

\[
\Delta_{i3}(t) = (g - \lambda)^2 - \frac{\lambda(L_i(t)) (|L_i(t)| - 1)}{\alpha(L_i(t)) + 1},
\]

where \(L_i(t)\) is the smallest, if possible non-trivial, group to which \(i\) belongs.

Groups arise 'by chance', and not until then, as \(|L_i(t)| > 1\), is this tension component relevant for group members. The value \((g - 1)^2\) assures non-negative tension values. Let \(w_{i3}\) be the relative importance for individual \(i\) of the group in comparison with the importance of the number of friends. We assume that \(w_{i2}\) and \(w_{i3}\) are equal for all individuals \((w_2, w_3 \geq 0)\). Accordingly, the total tension function becomes:

\[
\Delta_i(t) = |df_i - \sum_{j=1}^{f_i(t)} (1 - w_2 |x_i - x_j|) + w_3 ((g - 1)^2 - \frac{\lambda(L_i(t)) (|L_i(t)| - 1)}{\alpha(L_i(t)) + 1})\).
\]

This tension function is the basis of model 4 (Zeggelink 1997). In model 1, only the first issue is relevant (Zeggelink 1994). In model 2, the first two issues play a role (Zeggelink 1995). The first and third issue constitute the basis for model 3 (Zeggelink et al. 1996b).

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5 It should be realized that similarity also results from friendship because individuals tend to 'influence' each other such that they become more similar once they have become friends (Leenders 1995). We deal with this aspect in Section 4.
From the tension functions, behavior does not always follow straightforwardly because it is not always obvious for an actor how tension can best be minimized. Let us summarize both the main elements of the heuristic behavioral rules derived from the tension function and the additional assumptions. We assume that the number of friends is so important that \( a_i \) will never establish more friendships than the desired number of friends \( (df_i) \). Friendships, mutual choices, are always maintained. Once \( a_i \) has the desired number of friends, other individuals observe that \( a_i \) does not need any more friendships, therefore it is justified to assume that \( a_i \) will send a negative message to those individuals \( a_j \) that still try to initiate a friendship with him. If \( a_j \) receives a negative message from \( a_i \), \( a_j \) knows that tension will never be reduced by keeping a choice extended to \( a_i \) and replaces this choice. If at a certain moment in time \( a_i \)'s number of friendship choices is smaller than \( df_i \), \( a_i \) will add choices. For the addition of choices, \( a_i \) follows his preference order derived from the tension components of the similarity and group issues. For these bids he first selects those others who would provide him the largest tension reduction (thus in general, those individuals who choose him, are preferred). \( a_i \) makes as many friendship bids as would be necessary to complete his desired number of friendships, \( df_i \). If however, the difference between actual and desired number of friends is smaller than the number of preferred potential friends, i.e. more than one actor is ranked first in the preference order, \( a_i \) chooses randomly from them.

2.7. Learning

The only learning principle in these models is that an actor will never rechoose an actor that has sent a negative message to him. This actor has shown not be interested in a friendship, so keeping choices extended to that individual does not make sense.

2.8. Dynamics

The models are mainly deterministic. The single 'quasi-stochastic' element exists in the possible random choice when a number of equally preferred potential friendship candidates exists. Since actors 'observe' (by receiving negative messages) when they cannot establish any more friendships, an equilibrium situation in which no actor has an impetus to make any new choices, will always emerge with these behavioral rules. This situation will be characterized by the fact that all actors cannot lower their tension any further.

2.9. Variables

Independent variables, input parameters of the model, are population size \( g \), the distribution of desired numbers of friends \( df_i \) across the population, and the distribution of the dichotomous characteristic \( x_i \). Since the main goal is the explanation of structural characteristics of friendship networks, we have considered different dependent variables (Zeggelink 1993): mean number of friends per actor, dispersion of number of friends per actor, degree of transitivity, degree of segmentation (Baerveldt and Snijders 1994) and with respect to subgroup structure also the number of groups, their sizes, and their internal (\( \lambda \)) and external structure (\( \alpha \)).

2.10. Results

Main emphasis has been put on deriving relationships between the independent and dependent variables. With respect to empirical tests, models 1 to 3 have been tested using data on children in classes from grades 4 to 7 in the United States (among others: Hallinan & Kubitschek, 1990). We first derived hypotheses based on the associations between independent and dependent variables found in the simulations. Based on the limited number of data, partial support for the models was found. Subsequently, we used the characteristics of the classes as input data for our simulation models, and compared predicted and empirical network structures.
Model 2, where gender was the relevant dichotomous characteristic, seemed to be the best model for these data (Zeggelink 1993).

3. DECISION MAKING IN POLICY NETWORKS

3.1. Research topic

Before we explain how network evolution plays a role in decision making processes, we provide some background information on the type of decision making models in policy networks that we focus on. In the basic model, the Two Stage Model, network analysis is used to predict outcomes of policy making processes, where outcomes relate to decisions with regard to a number of different issues (Stokman and Van den Bos 1992). All actors in the policy domain have a preferred decision outcome, a policy position, with respect to the one-dimensional issues at hand. Actors differ in the amount of importance they attach to the different issues. They also differ in the amount of resources and their ability to vote. Public actors have voting power, private actors are actors without voting power. The only way in which private actors can affect decision outcomes, is by influencing other private and public actors. They exercise control over these other actors to establish that their positions get closer to their own positions. Such that as a result, when the public actors vote, the outcomes of the decisions are as close as possible to their own positions. However, an actor does not have the ability to influence all other actors. This is where the network comes into play. Influence from actor $a_i$ to $a_j$ can occur only if there is an access relation from $a_i$ to $a_j$. The amount of control of $a_i$ over $a_j$ depends on all other actors $a_k$ that have an access relation to $a_j$ and on the relative size of $a_i$'s resources in comparison to the $a_k$'s (including $a_j$).

The Two Stage principle now is based on the idea that decision making is a process that consists of two phases. In the first phase, actors attempt to influence others to whom access relations exist in the hope that their positions change in the direction of their own positions. Since all actors influence each other, it cannot be known beforehand whether influence attempts will be successful. The actual decision is taken in the second phase, when those actors with voting power vote for their position.

In the Two Stage Model of Stokman and Van den Bos (1992), the network is assumed to be given and constant. The only dynamic aspects concern the changes of positions as a result of the influence process. Data about the network of access relations, are obtained from interviews with experts in the field. In Stokman and Zeggelink (1996), Snijders et al. (1996), the network was made an endogenous component of the model, using the idea that actors try to establish the most useful access relations. The goal of this research project remains the prediction of decision outcomes, but now without using the information available on the access network. Nevertheless, this empirical network can be used to see to what extent the predicted network resembles the empirical one.

3.2. Actors and relationships

The closed set of $g$ actors $A = \{a_i | i = 1, ..., g\}$ is divided in public actors who have voting power, and private actors without voting power. Actors can be any social entity involved in the decision making processes varying from individuals to political parties and pressure groups. The relationship of interest is the directed access relation. A relation from $a_i$ to $a_j$ can exist only if $a_i$ requests access from $a_j$ and $a_j$ accepts this request.
3.3. Goals

The goal of an actor is that decision outcomes are as close as possible to his own policy positions, weighted by the importance he attaches to the specific issues at hand.

3.4. Characteristics

The first relevant actor attribute is voting power. Voting power of $a_i$ with respect to issue $f$ is constant over time: $v_{if} \geq 0, \sum_{i=1}^{k} v_{if} = 1$. Resources are also constant, and denoted $r_i$ for $a_i$.

The salience of issue $f$ for $a_i$ is denoted $s_{if}$ and also constant over time$^6$. If $m$ issues exist, $\sum_{i=1}^{m} s_{if} \leq 1$. The only non-constant actor attributes are policy positions. $a_i$'s position on issue $f$ at time $t$, his hypothetical vote if he would have to vote at this point in time, is denoted $x_{if}(t)$.

If an access relation exists from $a_i$ to $a_j$ at time $t$, $a_{ij}(t) = 1$. If no such access relation exists, $a_{ij}(t) = 0$. If $a_{ij}(t) = 1$, $a_i$ can exercise a certain amount of control over actor $a_j$. The potential control $c_{ij}(t)$ is defined as: $c_{ij}(t) = \frac{r_i a_{ij}(t)}{\sum_{k=1}^{k} r_k a_{kj}(t)}$.

3.5. Information

Actors are informed about the voting powers, resources, interests, and positions of all actors, and the total potential control of and over each actor. As a result every actor can compute the expected outcome $y_f(t)$ of decision $f$: $y_f(t) = \sum_{i=1}^{k} v_{if} x_{if}(t)$.

3.6. Behavioral rules

In terms of the network evolution part, this means that the goals of an actor are represented in his attempts to obtain access relationships with the aim to influence policy positions of others in a way that is beneficial to him.

When access relations have been established, $a_i$'s policy position in the next time period is the weighted mean of the position of $a_j$'s that have access to him (including himself). Actor $a_j$ exerts more influence over $a_i$, the more control $a_j$ has over $a_i$, and the higher the salience of $a_j$ with respect to issue $f$: $x_{if}(t+1) = \frac{\sum_{i=1}^{k} x_{if}(t) c_{ii}(t) s_{if}}{\sum_{j=1}^{g} c_{ii} a_{ij}(t) s_{if}}$.

Since no complete information exists, and since the actors are not capable to predict exactly the results of their influence attempts, they use heuristic rules when deciding which requests to make and which to accept. They try to make estimates of the consequences of their access relations in terms of outcome changes of issues salient to them. In deciding to which actors to make access requests, both the possible utility of such an access relation and the probability of success of actually establishing that relationship are taken into account.

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$^6$ Variable salience values are considered in Stokman and Stokman (1995).
In general, on each issue \( f \), actor \( a_i \) has a single-peaked utility function, depending on the distances between the outcome \( y_f(t) \) and his policy position \( x_f(t) \). The utility with respect to all \( m \) issues at time \( t \) is defined as: 
\[
U_f(t) = \sum_{f=1}^{m} s_f (y_f(t) - x_f(t))
\]

On the basis of this function, two different models, the Control Maximization (CM) model and the Policy Maximization model (PM), were developed that correspond to different underlying processes of individual behavior\(^7\). Since confrontation with empirical data has shown that the PM model produces the best predictions of decision outcomes, we deal with the PM model here.

The utility of a potential access relation to \( a_j \) from the viewpoint of \( a_i \) for one single issue is a function of \( a_i \)'s potential control over \( a_j \), \( a_j \)'s voting power, \( a_j \)'s potential total control over other actors, \( a_i \)'s salience with respect to issue \( f \), and a combination of \( a_i \)'s and \( a_j \)'s distances to the decision outcome (Stokman and Zeggelink 1996a). The sum over all issues is taken to obtain the expected total utility (\( EU_i(a_{ij}) \)) for \( a_i \) of an access relation to \( a_j \) and a corresponding preference rank for \( a_j \)\(^8\):

\[
EU_i(a_{ij}) = c_{ij}(t) \sum_{f=1}^{m} s_f (c_{jk}(t) - v_{jf}) (x_f(t) - y_f(t)) (y_f(t) - x_f(t))
\]

The probability of success depends on the rule actors use for accepting access requests. They attempt to avoid becoming accessed and thus influenced by other actors whose positions deviate from their own. Actor \( a_i \) therefore tends to accept requests of \( a_j \)'s whose policy positions are closest to his own policy positions. Based on the idea that other actors will apply the same rules, \( a_i \) assumes that the probability of success of establishing an access relation to \( a_j \) increases linearly with a decrease in overall policy distances to \( a_j \). On the basis of the expected utilities and probabilities of success, preference orders over all other actors are constructed. One for making access requests, one for accepting requests. Since policy positions and access relations change over time, so do preference orders\(^9\). A small degree of randomness appears in the model if ties exist in the preference order.

Actors are of course not allowed to establish access relations unlimitedly. Neither are they allowed to ignore all attempts from other actors that try to get an access relationship to them. Two very important restrictions are defined. The maximal number of allowed outgoing access relations is a function of the actor's amount of resources and the number of incoming access he accepted at the previous point in time. Similarly, the maximal number of incoming access relations an actor has to accept is a function of his voting power, resources and number of outgoing relations at the previous point in time.

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\(^7\) In the CM model, actors attempt only to obtain access relations to actors with high voting power and much control over other actors. At the same time, they have to accept incoming access requests from powerful actors (Stokman and Zeggelink 1996a).

\(^8\) For actors whose policy position is equal to the expected outcome on all issues, an exception to this function is made. \( c_{jk} \) refers to the total amount of control of \( a_j \) over all other actors.

\(^9\) When we consider the evolution of this network as a stage before influence processes come into action, the two stage model in fact becomes a three stage model. However, in general we consider the dynamics of both the relationship formation and the influence process simultaneously. This means that attractiveness of actors (in terms of their power or positions) varies over time.
3.7. Learning

Actors learn through experience. Actor \( a_i \) adapts his initial estimation of success with actor \( a_j \) every time \( a_j \) refuses to accept his access requests. The more often \( a_j \) rejects an access request, the lower \( a_i \) assumes the probability of success.

3.8. Dynamics

The models are mainly deterministic. The single 'quasi-stochastic' element exists in the presence of random choice when two actors are equally preferred. Since access relations are not always maintained, but can be dissolved when other actors become more attractive over time, a static equilibrium will emerge only with a small probability. Dynamic equilibria might emerge however. We generally restrict the number of iterations.

3.9. Variables

The relevant independent variables are the number of actors \( g \) involved, distributions of voting power \( v_{if} \), resources \( r_i \), salience \( s_{if} \), and initial policy positions \( x_{if}(0) \). Generally, outcomes of the decisions \( y_f \) are the dependent variables. However, since we predict the network, some network characteristics (like density) can function as dependent variables. We refer to (10) for more details on this aspect.

3.10. Results

The model performs rather well when it concerns predicting decision outcomes (empirical data). With respect to network density, the simulated network structures were closer to the empirical observed networks for the PM model than for the CM model. Nevertheless, the networks were still far off from the empirical networks (Stokman and Zeggelink 1996a, Stokman and Berveling 1996). Since the models pretend to model the formation of access relationships associated with the decision making process, a more detailed examination of the 'correctness' of the underlying behavioral rules and restrictions and assumptions seems appropriate. An important aspect here is that some model parameters always remain unknown, they cannot be dictated by theoretical arguments. Values for these parameters have been determined ad hoc. Recent developments attempt to solve this problem. A new method is used to estimate these parameter values so that network parameters predicted by the model correspond maximally to the empirical observations. By introducing statistical procedures within the simulation models, the parameter values do not have to be fixed, but can be adapted during the simulations. This was illustrated for the case of one parameter in Snijders et al. (1996a), and for the simultaneous estimation of more parameters in Snijders et al. (1996b).

4. FRIENDSHIP FORMATION AND SOCIAL INFLUENCE

4.1. Research topic

This research is an extension of the work described in Section 2 because it focuses on the emergence of friendship networks (Stokman and Zeggelink 1996b). However, since the model is some combined version of the models described in Sections 2 and 3, we deal with it here, but in a summarized version. The main aim of the model still is the explanation of emergence of structure in a friendship network from an initial situation of mutual strangers. However, we

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10 Moreover, in the latter extension, also random variation terms are included in the behavioral rules of the actors to account for the fact that in the explanation of the behavior of human beings, some variance always remains unexplained.
explicitly deal with the interplay between network dynamics and changing characteristics of individuals as a result of their friendships. As such, we deal with the fact that individuals may become less or more attractive as their characteristics change over time. They change as a result of social influence processes. Individuals do not only want to be friends with those similar to them, but friends also tend to become more similar. Moreover, we consider status effects and the possibility of dissolving friendships.

4.2. Actors and relationships

The actors are individuals. The relationship is friendship, defined as a mutual friendship choice. In contrast to the friendship relationship in Section 1, a measure of control, like that in Section 3, can be defined over the relationship. In contrast to the model of Section 2, negative messages are no longer necessary because the probability of success of actually establishing a friendship will be introduced.

4.3. Goals

The goals of an actor resemble those of Section 2. An actor wants to establish a desired number of friends. Now, he does not only want to be friends with actors similar to him, but also with higher status actors.

4.4. Characteristics

An actor has a desired number of friends (constant). Two comparison dimensions or issues and one aspiration dimension or issue (status) are defined (the extension to more issues is straightforward). Aspiration issues are introduced to include the fact that friendship formation can partly be determined on the basis of status differences. Aspiration issues imply a comparison between an actor's own aspiration level on the issue and the actual position of the (potential) friend on the issue. The aspiration issue is assumed constant and interval-scaled between 0 and 1. Comparison issues have the same function as the dichotomous characteristic of Section 2. They serve to determine similarity between actors. For now, the comparison dimensions are dichotomous. Other additional actor characteristics are similar to saliences in Section 3. Saliences are attached to the comparison and aspiration issues. Dependent on the model, they are constant or subject to social influence.

We mentioned earlier that a control value can now be defined over a friendship. This control value is determined analogously to the definition of control in Section 3, with the exception that resources are now dyad dependent. The smaller the difference between ai's actual value on aj's preferred value on the aspiration issue, the larger the resources of ai with respect to aj.

4.5. Information

Information aspects resemble those of Sections 2 and 3. An actor knows all characteristics of all actors in the population.

4.6. Behavioral rules

Similar to the estimations of the utility functions in Section 3, an actor can estimate the expected utility of a friendship with every other actor. The expected utility for ai of a (potential) friendship with aj is higher, the closer aj's values on aspiration issues are to ai's desired values, and the closer ai's desired values on comparison issues are to aj's. The weight of the distance on each issue depends on how salient that issue is for individual ai. Each actor also estimates the probability of success of establishing (or maintaining) a friendship with every
other actor. In general this depends on a number of aspects. If \( a_i \) already makes a friendship choice to \( a_j \) and \( a_j \) also chooses \( a_i \), the probability of success is estimated to be 0.9. If \( a_i \) chooses \( a_j \), but \( a_j \) does not choose \( a_i \), \( a_i \) lowers the probability of success with 0.1. If \( a_i \) does not choose \( a_j \), but \( a_j \) chooses \( a_i \), the probability of success is increased by 0.1. Initial estimates of probabilities of success are based on differences in resources.

Based on expected utilities and estimated probabilities of success, an actor constructs a preference order, and chooses as many candidates for friendship as his desired number of friends. After having made offers, friendships exist when choice is mutual. Probabilities of success are adapted, and social influence takes place. In principle this occurs in the same way as in Section 3 with the extension that not only positions, but also saliences may be subject to change. Since an actor's values and saliences on issues change as a result of their friendships, their and others' preferences orders change accordingly. Consequently friendships may well disappear again if one or both of the friends remove(s) his choice to the other as a result of more attractive alternatives.

4.7. Learning

Learning exists in the adaptation of estimated success probabilities.

4.8. Dynamics

The model has the combined dynamics of those in Sections 2 and 3. The process repeats itself until equilibrium, or after a predetermined maximal number of iterations.

4.9. Variables

Independent variables are the population size, the distribution of desired number of friends, and the values and saliences on the comparison- and aspiration dimensions. Dependent variables are network structural characteristics such as density and degree of segmentation, and the distribution of variable actor characteristics.

4.10. Results

Preliminary results indicate that the same initial distribution of saliences and values on comparison- and aspiration dimensions may result in a variety of network structures, just a few structures, or one and only one network structure. The final configuration of friendships and the effects of social influence depend on complex interactions between initial configurations and the presence or absence of social influence on saliences (Stokman and Zeggelink 1996b).

5. COOPERATION NETWORKS IN PRIMITIVE POPULATIONS

5.1. Research project

The goal of this research project is to contribute to a solution of the problem of how cooperation emerged in human social evolution. Contrary to models based on evolutionary game theory, our model aims to provide a potential explanation of the emergence of cooperation and at the same time, of group living. In contrast to other models of human social evolution, this model does not resort to a yet unexplained initial amount of cooperation (Boorman and Levitt 1980, Boyd and Richerson 1988). We start from 'scratch', that is, we formulate the problem in terms of a population in which individuals are organized in kinship groups and with kin directed altruism concentrated exclusively within these units. Such a population can be considered to
resemble a Pleistocene social organization of hunters and gatherers, in which cooperation is for a large part delayed exchange. This reciprocal altruism is mainly a characteristic element of human behavior, but difficult to explain. We hope to be able to explain the emergence of such patterned exchange relationships in an initially unrelated primitive population. We do so by assuming that the ecological conditions are such that survival (and reproduction) of actors varies stochastically over time and over actors (De Vos and Zeggelink 1994; Winterhalder 1986; Cosmides and Tooby 1992). This means that actors need to be and can be helped by others when in distress (when survival is at stake). By asking for and providing help, exchange relationships emerge.

5.2. Actors and relationships

The population consists of $g$ actors that represent kinship groups with kin directed altruism concentrated exclusively within them: $A = \{a_i | i = 1, ..., g\}$. The set is open in the sense that actors may die in the course of the process. The relationship of interest is an undirected exchange relationship. The more often exchange has taken place within that relationship, the stronger it is. Exchange relationships emerge from the process of asking for and providing help. Once $a_j$ has helped $a_i$, an asymmetric relationship from $a_j$ to $a_i$ exists. As soon as $a_i$ has also provided help to $a_j$, the relationship is mutual.

5.3. Goals

The goal of every actor is survival, and implicitly reproduction.

5.4. Characteristics

We first introduce time. Time is divided in discrete time periods $T_k$ subdivided in $t_{kl}$ intervals.

We distinguish two types of actors, social and asocial actors, corresponding to different behavioral rules or strategies (6). This characteristic is constant over time. So is the maximum number ($max$) actors that $a_i$ can help within one period $T_k$ (equal for all actors). Every actor $a_i$ can be in three different states: alive and in distress, alive and not in distress, or dead. Every $a_i$ has a time-dependent probability $q_i(T_k)$ of being in a state of distress ($q_i(T_0) = q_j(T_0)$ for all $i$ and $j$).

5.5. Information

An actor is initially informed only about the number of other actors in the population. No information exists on their characteristics. These may however be learnt over time (see later).

5.6. Behavioral rules

An actor $a_i$ who is in distress at the beginning of period $T_k$, can survive if and only if he is supported by another actor $a_j$ within the same period. If $a_j$ supports, he performs an act of (reciprocal) altruism, that is, the act involves costs, and produces in $a_j$ the intention to call in $a_i$'s assistance at a future moment if fortunes are reversed. An actor can ask one and only one actor for help in every interval of a time period\(^{11}\). If $a_i$ does not receive any support, he dies.

An actor $a_j$ who is asked to support another actor $a_i$ cannot provide support if he himself is in distress. If he is not in distress, he can provide support to maximally $max$ actors $a_i$ within one

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\(^{11}\) An actor will never receive support if he does not ask for it because we assume that actors will never take the initiative to provide support by themselves.
The costs of helping another actor are expressed in an increase by a factor \( c \) of the probability \( q_j(T_{k+1}) \) in the next period only (\( q_j(T_{k+1}) = q_j(T_k) + c \)), meaning that the act of supporting, temporarily increases the probability to get into a state of distress.

The behavioral rules that describe which actors an actor will ask for help when in distress, and which actors an actor will help when asked for, differ for social and asocial actors.

A social actor \( a_i \) keeps a list of all actors \( a_j \) whom he has provided support or has received support from in the past \((r_{ij} = \text{the number of times he has received support from } a_j; p_{ij} = \text{the number of times that he has provided support to actor } a_j\)) If a social actor \( a_i \) is in distress, he asks other actors for support in order of \((p_{ij} - r_{ij})\). A number of actors \( a_j \) may exist for whom these differences are equal. If the social actor has a so-called 'prefer-old-helping-partner'-trait, he prefers to ask that \( a_j \) for whom \( r_{ij} \) is highest. Without this trait, he chooses randomly among these equally preferred actors.

If a social actor \( a_j \) is asked for support, he cannot provide it when he is in distress himself or when he has already provided help up to \( \text{max} \) other actors in the current time period. If he does not happen to be in any one of these situations, he provides support to \( a_j \) if \( r_{ij} - p_{ij} \) is not smaller than -1. This means that an actor will support another actor for a second time, only if that actor has returned help in between. If more actors \( a_i \) ask for support simultaneously, \( a_j \) gives priority to these requests according to the following rule (about the reverse of the rule according to which support is asked): provide support in order of \((r_{ij} - p_{ij})\), as long as this difference is not smaller than -1, or until \( \text{max} \) is reached. So priority is given to those \( a_i \) for whom the difference between the number of times received and given support is highest. In case \( a_j \) is asked for support by two or more \( a_i \)'s for whom these differences are equal, he prefers those \( a_i \) for which \( p_{ij} \) is highest if he has the 'prefer-old-helping-partner'-trait. He feels "obliged" to those actors with whom he shares a common history of exchanges, although he is just as even with them as he is with strangers. If he does not have this trait, he chooses randomly among those ranked equally.

An asocial actor \( a_i \) keeps a list of all actors \( a_j \) from whom he has received help in the past, and also registers the number of times they have helped him. In case of distress, he asks for support from \( a_j \) in increasing order of \( r_{ij} \). Thus, if possible, he first asks an actor who has never helped him before. If several actors \( a_j \) have provided support an equal number of times, \( a_i \) chooses randomly from those \( a_j \)'s. In cases the asocial actor \( a_i \) is asked for support, he never supports it.

5.7. Learning

Actors learn a little of the strategies of the other actors over time on the basis of these latter's responses when asking for help. They know for example that an actor cannot be asocial once he has provided support. The reverse does not hold. An actor may have been unwilling to give support for other reasons than his being asocial.

5.8. Dynamics

The model is partly stochastic and partly deterministic. It is stochastic in the sense that the probability to get in distress determines which actors will need help. This probability varies

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12 This is for instance always the case if \( a_i \) happens to be in distress in the initial situation, in which both \( p_{ij} \) and \( r_{ij} \) are zero for all \( a_j \). But in later time periods it may also happen that for two or more \( a_j \)'s the difference is the same.

13 The asocial individual is capable of begging for support and knows that asking the same person twice is unsuccessful, but he does not see a connection with reciprocity.
over time as a result of providing help to other actors. The unique equilibrium of the process is that all actors have died (when no reproduction occurs)\textsuperscript{14}. Therefore, we assume a maximum number of periods. Nevertheless all actors may still die before this time.

5.9. Variables

Relevant independent variables are among others, the initial number of actors $g$ in the population, the initial probability to get in distress ($q_i(T_0)$), the costs of providing help ($c$), and the initial proportion of social actors. Dependent variables of interest are the total number of survivors, the proportions of social and asocial actors in the end population (and thereby survival probabilities), and the network structure. In particular we have examined the degree of segmentation (subgroup structure) in the network (De Vos and Zeggelink 1994).

5.10. Results

The model has been used to investigate the viability of social and asocial strategies of giving and requesting support. We examined whether social strategies with a 'prefer-old-helping-partners' trait are viable in competition with asocial (cheating) strategies. The results suggest that harshness of conditions and size of the initial population are the main determining factors of the viability of a kind of reciprocal altruism that includes a "prefer-old-helping-partners" trait. Furthermore, we analyzed the network structure in the population of surviving actors. No empirical tests of the models prevail (De Vos and Zeggelink 1994, 1996).

6. COALITION FORMATION IN POLITICAL NETWORKS

6.1. Research topic

In most western European countries, usually three or more political parties gain parliamentary representation. It seldom happens that one party controls the majority by itself, and coalition governments have to be formed, which together control a majority in the parliament. Our goal is to study the effects of changes in institutional and structural parameters of the coalition formation process, on the final outcome (Van Roozendaal and Zeggelink 1994, 1996). Structural characteristics are the number of parties, the number of relevant ideological dimensions, the weights of the parties and the distribution of parties on the dimensions. Institutional rules describe whether all parties are allowed to act simultaneously in the formation process or whether one party receives a mandate to lead the process.

6.2. Actors and relationships

The parliament constitutes the set $A = \{a_i \mid i = 1, \ldots, g\}$. As such, actors are parties or coalitions of parties. Coalitions, or proto-coalitions, that are formed, are considered to be new actors. These new actors are subsets of the initial set of actors. The size of the set of actors decreases over time when (proto) coalitions are formed. The relationship of interest is the mutual willingness to form a coalition. This mutual willingness appears from choices towards each other, representing the preference to be in a coalition with the receiver of the choice. Once choice is mutual, the two parties constitute a coalition.

6.3. Goals

The goal of every actor is to be in the majority coalition that accounts for minimal policy distance between the actor's position and that of the coalition.

\textsuperscript{14} Thus in fact, the model is incomplete without any reproduction processes.
6.4. Characteristics

All actor attributes are constant. Let $w_i$ be the weight of actor $a_i$. It indicates its voting power, the number of parliamentary seats. Assume that $m$ one-dimensional policy dimensions exist. Each actor $a_i$ has a position $p_{ik}$ on dimension $k$, $k = 1, \ldots, m$. This position indicates the most preferred position of the actor on that dimension. We assume that actors have single-peaked preference functions.

A coalition is a set of actors $C = \{a_i \mid i = 1, \ldots, z\}$. $C$ is a winning coalition, a majority coalition, if and only if its weight, the sum of the weights of the actors that are included in $C$, reaches a certain prescribed majority criterion $q$: $\sum_{i=1}^{z} w_i \geq q$, where $q$ is the smallest integer value exceeding half of the total weight.

The policy position of a coalition $C$ on dimension $k$, $p_{ck}$ is the weighted policy position of the actors that constitute the coalition: $p_{ck} = \frac{\sum_{i=1}^{z} w_i p_{ik}}{\sum_{i=1}^{z} w_i}$.

If we assume that all dimensions are equally weighted, the distance between an actor $a_i$ and coalition $C$ is: $D_{i,C} = \sum_{k=1}^{m} |p_{ck} - p_{ik}|$.

6.5. Information

Each actor knows the position(s) $p_{ik}$ and number of seats $w_i$ of all other parties or coalitions in the population.

6.6. Behavioral rules

Coalition formation is modelled as a process of proto-coalition formation in two different ways: In the Basic Model (BM) each actor $a_i$ is allowed at any point in time to make one and only one choice toward an other actor. From the goals formulated earlier, it follows that the smaller the policy distance between $a_i$ and $a_j$, the higher $a_j$ is ranked on $a_i$'s preference list. When more actors are ranked first, $a_i$ chooses randomly from those actors. Consequently, no overlapping proto-coalitions can be formed and two possible next steps exist.

- When mutual choices exist, a proto-coalition is formed as a new actor that replaces the two constituting actors. If this proto-coalition is winning, the process comes to an end because a majority coalition has emerged. If the proto-coalition is not winning, the process restarts with the new (smaller) set of actors.

- When no mutual choices exist, one randomly chosen actor will withdraw its present choice and replace it, if possible, by a choice towards an actor that chose it. As a result, a proto-coalition is formed, and the process continues as described above.

In the Formateur Model (FM), one actor, the formateur gets the mandate to start forming coalitions. This model is similar to the BM with the exception that only the formateur takes the initiative, by choosing potential coalition candidates. The actor that receives the choice then decides whether it wants to be in the coalition with the offering party or coalition or not. If it
does, a new proto-coalition is formed. If not, the formateur continues looking until a majority coalition has been formed, or until all actors have refused participation in a coalition.

The major difference between BM and FM is that minority coalitions may emerge with the latter.

6.7. Learning

The learning principle is that actors do not choose actors again once they have refused to be in a coalition together.

6.8. Dynamics

Our model differs from a number of sequential models of coalition formation in the sense that it is not completely deterministic. At certain stages in the formation process it may contain 'quasi-stochastic' elements when a random choice has to be made.

6.9. Variables

We use the following independent variables: the number of parties $g$ in the parliament, the number of policy dimensions $m$, the distribution of policy positions $p_{ik}$, and the distribution of weights $w_i$. For the FM we also analyze different types of formateurs (largest, central, or random party). We studied the effects on the probability that a majority coalition emerges. If the resulting coalition is a majority coalition, we examined effects for both models on the following dependent variables: the number of parties in the coalition, the total weight of the coalition, and the policy heterogeneity on each dimension of the coalition.

6.10. Results

Interesting results can be derived from the simulations, but so far no empirical tests of the derived hypotheses prevail. We find for instance that the number of parties in the parliament has a negative effect on the relative number of parties in the coalition. A normal weight distribution of the actors or the presence of a specific formateur also leads to a lower number of actors in the coalition. The presence of a specific non-random formateur however leads to higher coalition weights and higher degrees of coalition heterogeneity.

7. DISCUSSION

We have presented an overview of different actor oriented models of network evolution that have been developed in the last couple of years. The models were constructed in different fields of application but all have in common that the emergence of network structure is directly or indirectly of interest. Each model is based on a set of actors and a set of behavioral rules of these actors, resulting in interaction mechanisms and the coming into existence of some network pattern of relationships. Actors vary from individuals and families to political parties, and this set may be either closed or open. When it is closed, no actor leaves or enters the population. In an open set of actors, actors either disappear, or new actors emerge as the merge of other actors. Relationships are either directed or undirected and vary from friendship to cooperation, and access to coalition partners.

In most models, choice of partners is deterministic and based on preferences determined on the basis of characteristics of the potential candidates. In some of the models, the importances or
saliences of different dimensions of actor attributes are taken into account. If they are present, they may be subject to social influence, just like variable actor attributes may be.

Usually if an actor has to choose from a set of actors that are equally preferred for a relationship, choice is random. This causes the same initial configuration of actors and characteristics to result in a variety of networks.

Simulation is used to obtain distributions of possible resulting network structure because this and other aspects of the models, make it hard to be solved analytically. We think that the use of these kind of simulation models, by examining the influence of both endogenous and exogenous variables, contributes to the improvement of theory building. Hypotheses can be derived and tested at the empirical level. This becomes even easier when the models are made better suitable for empirical testing. When we construct the models in an even more explicit theoretical manner, by including a random element that captures non-explained variance in the empirical data, statistical testing will be easier to accomplish. Such unexplained variance can represent incompleteness of data gathered, incompleteness of the theoretical model or real stochastic behavior of the model. This could be stochastic behavior of the actors in the networks, when the random component is included in the behavioral rules (like in random utility models applied in econometrics). Such models more have the pretension to be direct representations of a realistic network development over time. The model's assumptions on individual behavior can be tested and parameter values in the models can be estimated. When enough empirical data are available, it can also be examined whether parameter values differ in different populations. This could for example represent that the friendship formation process among children differs from that among adults. For a more extensive introduction into such models, see Snijders (1996) and Snijders et al (1996a, b).

Zeggelink et al (1996b) had already shown that the link between the micro level where actors act and the macro level where phenomena occur as a consequence and a cause of these actions can be modelled in a straightforward way when using object oriented models. Here we have shown the principles of such models in different fields of applications. The models, as they were presented here, were simple illustrations. General extensions to more issues or actor attributes exist or are easily incorporated. A next step is the integration of the different models in one framework such that the combination of aspects in the different applications leads to the ongoing development of new models. By doing so, the typical advantages that exist in the different models can be transformed to all other models in a straightforward manner if necessary. Moreover, correspondences and differences in behavioral rules can be examined across the different fields of applications. As a result, not only can different theoretical or fundamental questions be answered within one framework, but different micro theories can be tested against one another.

To conclude, we agree with Stokman and Doreian (1996) when they say that dynamics and evolution of networks are topics that need to be addressed in social network research. They notice that the social network field is well enough developed now to deal with these kind of subjects. New methods and techniques need to be developed to address question of emergence and existence of social networks. Here we have illustrated one way of modeling network evolution that includes, and can include, the different elements suggested by Stokman and Doreian: the definition of a goal structure of the network members and the instrumental character of the network, the information process, parallelism of actions, reference to empirical work in order to estimate parameters in the models and to determine goodness of fit.
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