THE PICRATE MODEL FOR FITTING THE AGE PATTERN OF FIRST MARRIAGE

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RÉSUMÉ – Ajustement du schéma par âge d’entrée en première union : le modèle PICRATE
Nous présentons un nouveau modèle qui décrit le schéma par âge d’entrée dans une nouvelle situation. L’introduction du taux de recrutement rend l’utilisation de ce modèle très simple, en particulier pour simuler l’évolution d’une population au cours du temps. Le modèle est défini par trois paramètres : a₀, l’âge minimal à l’événement, p_max, la proportion maximale de la population qui connaîtra l’événement, et r_max, le taux maximal de recrutement. Ce modèle est testé sur l’entrée en premier mariage, et est comparé au modèle de Coale-McNeil. Nous avons appliqué ce modèle au cas de la Zambie, ainsi qu’à divers pays africains ayant des schémas différents. Le modèle peut s’appliquer à divers processus, tels que les premiers rapports sexuels, la première naissance, le premier emploi, la première migration adulte, etc.

MOTS-CLÉS – Afrique, Âge au premier mariage, Analyse biographique, Enquête EDS, Modèle de Coale-McNeil, Modélisation mathématique

SUMMARY – We present a new model for fitting the age pattern of entry into a new situation. The formulation of the marriage recruitment rate is simpler than in other available models, and particularly useful for computer simulation of the time-evolution of a population. The model has three parameters: a₀, the starting age at event, p_max, the maximum proportion of the population at risk p(a), and r_max, the maximum value of the recruitment rate r(a). This model was tested on the entry into first union, and was compared with the widely-used Coale-McNeil model. We applied this model to the case of Zambia, and to a variety of African countries with different features. This model could also be used for fitting a variety of processes, such as first sexual intercourse, first birth, first job, first adult migration, etc.

KEYWORDS – Africa, Age at first marriage, Coale-McNeil model, DHS survey, Event history, Mathematical model

1. INTRODUCTION

Processes describing an entry into a new situation are very general, and can be found in various domains such as demography, economics, sociology and epidemiology. This is the case, for instance, of first sexual intercourse, first marriage, first birth, first job, first adult migration. These processes have common features: they start at a given age, are concentrated within a relatively narrow age group, are cumulative, and concern only a fraction of a population. The proportion of persons who have experienced the event at age (x) follows a sigmoid function, that can be modelised for a variety of purposes:

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smoothing data, comparing populations, calculating mean and variance of age at event, and computer projections and simulations.

The model that we are presenting here was first applied to the case of first marriage, but has a much broader scope. This research came out of computer modelling of the dynamics of the HIV-AIDS epidemic in Southern Africa. This exercise required functions that model the age-specific “recruitment rate” into first marriage in a cohort, a common indicator in demography. The recruitment rate \( r(a) \) determines the first marriage rate \( f(a) \) and hence the whole distribution of ages at marriage.

Several models were already available, although all required complex calculations. The most widely used model in demography is that of Coale and McNeil (1972), who presented a formula in closed form for a female cohort. This model has an empirical dimension, since it is based on the age pattern of first marriage in 19th century Sweden. Another formula in closed form had been proposed at about the same time by Hernes [1972], though with more problems for practical applications. The Coale-McNeil model was further analysed and improved by a variety of authors [Feeney, 1972; Ewbank, 1974; Rodriguez and Trussell, 1980]. The Coale-McNeil model is based on a double exponential, which has been recently re-analysed, simplified and made more systematic by several authors [Kaneko, 1991; Liang, 2000; Kaneko, 2003]. In the latest formulation by Kaneko (2003), the model became a generalised Log-Gamma function, still requiring complex calculations for practical purposes.

The aim of this paper is to provide a simpler model, fitting empirical data at least as well as the Coale-McNeil reference model, independent on any empirical pattern, and easier to program for computer simulations. In section 2, we provide the reader with background information on other models. In section 3, we present the Picrate model. In section 4 we compare our approach with the Coale-McNeil model. In section 5 we present an empirical application on a case study.

2. BACKGROUND

The model developed by Coale and McNeil was based on a previous finding by Coale [1971] that a standard function, when scaled and shifted, was a good fit to the age patterns of first marriage across a range of populations. The Coale-McNeil model is defined by a probability density function composed of a double exponential (see below section 4). This model implies that the distribution of waiting times is composed of a series of exponentially distributed waiting times. In the original model, this was interpreted as a series of waiting times as in real life, such as: time to entry into marriageable state, time to meet a possible mate, time for engagement, and eventually waiting time for marriage.

More generally, when a process is controlled by a constant rate \( r \), the mean time for the event to happen is simply \( T = 1/r \). Applied to first marriage, such a simple model where the recruitment rate \( r_c \) that commences at age \( a_0 \), is constant, results in the proportion ever-married given by \( p(a) = p_{\text{max}} \left[ 1 - \exp \left( -r_c (a - a_0) \right) \right] \). Although this provides a good fit to data for later ages, it fails to model the initial exponential rise that is observed. We discovered that a much better fit could be obtained if the age-specific recruitment rate \( r(a) \) was phased-in over a time scale \( T \), from 0 at age \( a_0 \) to a maximum value \( r_{\text{max}} \). Its rate of change is defined to be \( dr/da = (a - a_0)(r_{\text{max}} - r)/T^2 \). We took...
\(dr/da\) as a Weibull distribution function, and it turned out that a good fit was obtained when \(T = 1/r_{\text{max}}\), which has the great advantage of reducing the number of parameters by one. This was the rationale for our new model. The acronym PICRATE describes this choice: Phased-In-Constant Rate.

3. DESCRIPTION OF THE PICRATE MODEL

We define the following quantities for an age-cohort of women:

- \(a\): age (continuous, measured in years)
- \(a_0\): age at which marriages begin
- \(p(a)\): proportion ever married at age \(a\).
- \(p_{\text{max}}\): proportion that will eventually marry (maximum value of \(p\)).
- \(r(a)\): age-specific first marriage recruitment rate (nuptiality rate among susceptible women).
- \(r_{\text{max}}\): maximum value of \(r(a)\).
- \(f(a)\): first marriage rate, or marriage schedule, the proportion of the cohort that marries per time unit. In a population, \(f(a)\) is equal to \(dp/da\), and in a stable population to \(dp/da\). The \(f(a)\) function is the probability density function of first marriages.

The ever-married proportion \(p(a)\) increases from 0 at \(a_0\) to a maximum value \(p_{\text{max}}\), and \(r(a)\) rises from 0 at \(a_0\) to a maximum value \(r_{\text{max}}\). At a given age, the proportion of the cohort that has yet to marry is \((p_{\text{max}} - p)\). Before \(a_0\), the functions \(r, f,\) and \(p\) are strictly zero.

The model’s equations are described here in scale-independent “standard” form, by defining the dimensionless standardised age as

\[
x = r_{\text{max}}(a - a_0)
\]

(1)

where \(T = 1/r_{\text{max}}\) is the time-scale for the process. We also defined normalised functions as:

\[
\hat{p}(x) = \frac{p(x)}{p_{\text{max}}}, \quad \hat{f}(x) = \frac{f(x)}{p_{\text{max}}} \quad \text{and} \quad \hat{r}(x) = \frac{r(x)}{r_{\text{max}}}.
\]

(2)

The first two functions are respectively the cumulative probability distribution function and the probability density function.

The rate of new marriages for \(a > a_0\) (or \(x > 0\)) is given by:

\[
\frac{dp}{dx} = \hat{r}(1 - \hat{p}),
\]

(3)

and we set

\[
\hat{r}(x) = 1 - \exp\left(-x^2/2\right).
\]

(4)
Consequently
\[ \frac{d\hat{r}}{dx} = x \exp\left(-\frac{x^2}{2}\right), \] a Weibull distribution function. \hfill (5)

The choice of a Weibull function is justified by the sigmoid shape of the recruitment rate in empirical data. Other functions which are more symmetrical, such as the Logistic function, would not have permitted to fit the empirical data. Furthermore, the Weibull function is widely available, and easy to calculate numerically [Weibull, 1951].

From equations (3) and (4) we derive an explicit formula for \( \hat{p}(x) \) by separation of variables and integration to obtain:
\[ \hat{p}(x) = 1 - \exp\left(-\int \hat{r}(u) \, du\right). \] \hfill (6)

We evaluate the integral:
\[ \int_0^x \hat{r}(u) \, du = x - \int_0^x \exp\left(-\frac{u^2}{2}\right) \, du \] \hfill (7)

to obtain a final expression for \( \hat{p}(x) \):
\[ \hat{p}(x) = 1 - \exp\left(-x + I(x)\right). \] \hfill (8)

where
\[ I(x) = \int_0^x \exp\left(-\frac{u^2}{2}\right) \, du = \sqrt{2\pi} \left[N_C(x) - \frac{1}{2}\right], \] \hfill (9)
and \( N_C(x) \) is the cumulative normal distribution function with mean 0 and standard deviation 1.

Equation (9) is not a formula in closed form, but it is nevertheless easily evaluated with the use of special functions that are provided by most mathematical computer packages. For instance, Abramowitz and Stegun [1964] provide the following nearly single-precision (error < 1×10^{-5}) approximation for \( N_C(x) \), where we first set \( t = 1/(1+0.33267 x) \), and then
\[ N_C(t(x)) = 1 - (0.4361836 t - 0.1201676 t^2 + 0.9372980 t^3) \exp(-t^2/2)/\sqrt{2\pi}. \] \hfill (10)

Other related special functions may be used in place of \( N_C(x) \).

Furthermore, the function presented in (9) can be integrated numerically, to obtain the median, mean and variance in units of standardised age: \( MED(X) = 1.8693 \), \( E(X) = 2.090 \), \( V(X) = 1.292 \); the standard deviation is then \( \sigma(X) = 1.137 \). These estimates can be used to calculate directly, in units of years and years^2, the mean and variance of the distribution of age at first marriage, simply by inverting the change in variables:
\[ MED(A) = a_0 + \frac{1.8693}{r_{\text{max}}}. \]
\[ E(A) = a_0 + \frac{2.090}{r_{\text{max}}} \]  
\[ V(A) = \frac{1.292}{r_{\text{max}}}^2. \]  

Finally, directly from (3), (4) and (8), we obtain:

\[ \hat{f} = \frac{dp}{dx} = \left(1 - e^{-\frac{1}{2}x^2}\right) \exp\{-x + I(x)\}. \]  

Straightforward derivations provide the needed functions after the change in variable is inversed from (1). Expanding from the normalised functions and standardised age, we have the marriage recruitment rate from (4):

\[ r(a) = r_{\text{max}} \left\{1 - \exp\left[-\frac{1}{2}r_{\text{max}}^2(a-a_0)^2\right]\right\} \]  

the schedule of first marriages from (13):

\[ f(a) = \frac{dp}{da} = p_{\text{max}} r_{\text{max}} \left\{1 - e^{-\frac{1}{2}r_{\text{max}}^2(a-a_0)^2}\right\} \exp\{-r_{\text{max}}(a-a_0) + I(r_{\text{max}}(a-a_0))\} \]  

and the ever-married proportion from (8):

\[ p(a) = p_{\text{max}} \left\{1 - \exp\left[-r_{\text{max}}(a-a_0) + I(r_{\text{max}}(a-a_0))\right]\right\} \]  

The curve for \( p(a) \) describes the history of first-marriages in the cohort, but for a stable population it also describes the age-specific ever-married proportion for a population. Note that the curve is determined by the three parameters \( a_0, r_{\text{max}} \) and \( p_{\text{max}} \).

4. THEORETICAL COMPARISON WITH THE COALE-MCNEIL MODEL

In this section we present a simple formulation of the Coale-McNeil model – including the recruitment rate – based on the original age-distribution function given by Coale and McNeil [1972], a modified version by Rodriguez and Trussell [1980], and an expression for the cumulative age-distribution given by Kaneko [2003]. We then formally compare the PICRATE and Coale-McNeil models.

The Coale-McNeil [1972] distribution of first marriages (or marriage rate) in a female cohort is given by the following formula:

\[ \hat{f}_{\text{CM}}(a) = \frac{\beta}{\Gamma(\alpha/\beta)} \exp\{-\alpha(a-\mu) - \exp\{-\beta(a-\mu)\}\}. \]  

where \( \Gamma(\theta) \) is the gamma function. Note that this is the distribution of ages at first marriage, excluding those who will never marry. The age-specific schedule (proportion of the entire cohort that marries at a given age) is given by \( f(a) = p_{\text{max}} \hat{f}(a) \).
We define the following notation:

\[ \theta = \alpha/\beta \quad \text{and} \quad z(a) = \exp\left[-\beta(a-\mu)\right]. \]  

(18)

Then (17) may be written as

\[ \hat{f}_{CM}(a) = \frac{\beta}{\Gamma(\theta)} z^\theta e^{-z}, \]  

(19)

and the cumulative distribution as

\[ \hat{p}_{CM}(a) = 1 - \frac{\gamma(\theta,z)}{\Gamma(\theta)}, \]  

(20)

where the lower incomplete gamma function is defined here as

\[ \gamma(\theta,z) = \int_0^z e^{-u} u^{\theta-1} du. \]  

(21)

It is easily verified that (20) is the cumulative distribution function by differentiating:

\[ \hat{f}_{CM}(a) = \frac{d\hat{p}_{CM}}{da} = \frac{-1}{\Gamma(\theta)} \frac{dz}{da} \frac{d\gamma}{d\theta} = \frac{\beta}{\Gamma(\theta)} z^\theta e^{-z}. \]

Finally, the Coale-McNeil first marriage recruitment rate (or hazard) is given by

\[ r_{CM}(a) = \frac{\hat{f}_{CM}(a)}{1 - \hat{p}_{CM}(a)} = \frac{\beta z^\theta e^{-z}}{\gamma(\theta,z)}. \]  

(22)

It is straightforward to show that as \( a \to \infty \), \( r_{CM} \to \alpha \). So, as in the PICRATE model, the Coale-McNeil first marriage recruitment rate tends asymptotically to a constant value at late ages, after an initial exponential rise.

We further compared formally the PICRATE model with the Coale-McNeil model. The Coale-McNeil age-distribution \( \hat{f}_{CM}(a) \) is given by (17). A standard two-parameter version of this model was provided by Coale and McNeil [1972], but for this comparison between the models we used the adjusted standard version by Rodriguez and Trussell [1980]. For mean \( \mu_c \) and standard deviation \( \sigma \), for which we also define \( k_c = 1/\sigma \), we define standardised age as

\[ y = \frac{(a-a_c)}{\sigma} = k_c(a-a_c). \]  

(23)

We then set \( \alpha = 1.145 \), \( \beta = 1.896 \) and \( \mu = -0.805 \), to obtain

\[ \hat{f}_s(y) = (1.2813/\sigma) \exp\left[-1.145(y + 0.805) - \exp\left(-1.896(y + 0.805)\right)\right]. \]  

(24)

The comparison between the Coale-McNeil and the PICRATE models was done by computing the Coale-McNeil functions (19), (20) and (22) for the mean and standard deviation of the PICRATE distribution: \( \mu = 2.090 \) and \( \sigma = 1.137 \), in units of PICRATE...
standardised age $x$ (1). The PICRATE functions plotted were (4), (8) and (13). Special functions were computed by algorithms in Press et al [1992].

The curves are plotted in Figure 1 below. The maximum absolute difference between the age-distributions was 0.015, between the cumulative distributions 0.004, and between recruitment rates 0.02. (Note that in this comparison, PICRATE $r_{max} = 1$, and C-M $r_{max} = \alpha/\sigma = 1.007$.) The two distributions match closely, and since each model may be scaled and located with three parameters, the PICRATE model serves as a close approximation and hence as an alternative to the Coale-McNeil model.

![Figure 1](image-url)

**FIGURE 1.** Theoretical comparison of the PICRATE model with the Coale-McNeil model. Values of $r$, $p$, and $f$ are plotted against standardised age $x$, with the PICRATE curves as solid lines, and the Coale-McNeil curves as dotted lines.

5. EMPIRICAL VALIDATION OF THE PICRATE MODEL

We have applied the PICRATE model to the case of Zambia, and compared it to the Coale-McNeil model. Empirical data were provided by the Demographic and Health Surveys (DHS), conducted in 1992, 1996 and 2001, which were combined for increasing sample size and reducing random fluctuations. These data provided at the same time the proportion ever-married and the age at first marriage, which allowed us to compute the empirical values of $p(a)$ and $f(a)$. Fitting the PICRATE model was done by minimising the squared-distance between the empirical $p(a)$ and the model $p'(a)$, using the Solver module in Excel, and a similar procedure was used for the Coale-McNeil model.

Results give the following values for the PICRATE model: $a_0 = 11.697$, $r_{max} = 0.298$, and $p_{max} = 0.988$, which results in a mean age at marriage = 18.72 years, and a standard deviation = 3.82 years. Comparison with the empirical values, for $p(a)$ and $f(a)$, are displayed in Figure 2, and shows the goodness of fit of the model. The proportion ever married fitted closely the empirical data, except below age 14 and above...
age 40, both differences which could be explained by sample size fluctuations (less than 5% of marriages occurred prior to age 14, and only 21 marriages occurred above age 35 out of a sample of 22739 women), and possibly by cohort changes or by misreporting of age at first marriage. For the marriage rate, the fitting was also good, including at young ages (below age 20). The age pattern was well described by the PICRATE model, although empirical values tended to be somewhat below the fitted curve between age 20 and age 30 and somewhat higher from age 30 to age 40 (cf. Figure 2).

The Coale-McNeil model fitted equally well the Zambian data, and was always very close to the PICRATE model (cf. Figure 2). Discrepancies with $p(a)$ were the same at very young and very old ages, as well as discrepancies with $f(a)$.

We also tried to use other models on the Zambian case. The Kaneko model (the Log-Gamma function) did not fit the $p(a)$ function any better. In addition, it tended to over-estimate the mode of the $r(a)$ function around age 20, and tended to decline faster than the other models afterwards, leading to an increasing bias above age 30. The Hernes model was much worse, since it tended to produce a very symmetrical $f(a)$ function and missed the sharp increase in the $p(a)$ function at young ages.

![Figure 2. Fitting empirical data with the PICRATE model and comparison with Coale-McNeil model, Zambia DHS surveys, female population](image-url)

6. APPLICATION TO AFRICAN DATA

To verify how the model behaved in a variety of situations, we applied the PICRATE model to three case studies using African DHS datasets: one with very low age at marriage (Niger, 1992), one with average age at marriage (Tanzania, 1999), and one with very high age at marriage (Botswana, 1988), which represent the maximum variations in age pattern of first marriage from African data.

Table 1 and Figure 3 show the goodness of fit with the two models. Distances to empirical data were basically the same in the three cases, and estimates of mean age,
median age, and variance in age at marriage were also similar. There is therefore no
doubt that the PICRATE model performed well, and at least as well as the Coale-
McNeil model, for fitting African data on age at first marriage.

TABLE 1. Fitting the PICRATE model, and comparison with the Coale-McNeil
model in countries where first marriage is early (Niger), average (Tanzania), and late
(Botswana). The Coale-McNeil parameters below are related to the parameters in (17)
by $\alpha = 0.174/k_{CM}^{CM}$, $\beta = 0.2881/k_{CM}^{CM}$, and $\mu = a_{0}^{CM} + 6.06 k_{CM}^{CM}$.

<table>
<thead>
<tr>
<th>Country</th>
<th>Niger</th>
<th>Tanzania</th>
<th>Botswana</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year of survey</td>
<td>1992</td>
<td>1999</td>
<td>1988</td>
</tr>
<tr>
<td>Coale-McNeil parameters</td>
<td>$a_{0}^{CM}$</td>
<td>11.165</td>
<td>12.784</td>
</tr>
<tr>
<td></td>
<td>$k_{CM}^{CM}$</td>
<td>0.420</td>
<td>0.601</td>
</tr>
<tr>
<td></td>
<td>$P_{max}$</td>
<td>0.999</td>
<td>0.986</td>
</tr>
<tr>
<td>Coale-McNeil results</td>
<td>Mean age</td>
<td>15.44</td>
<td>19.10</td>
</tr>
<tr>
<td></td>
<td>Median age</td>
<td>14.38</td>
<td>17.87</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>7.65</td>
<td>15.57</td>
</tr>
<tr>
<td></td>
<td>Distance from empirical data</td>
<td>0.0109</td>
<td>0.0028</td>
</tr>
<tr>
<td>PICRATE parameters</td>
<td>$a_{0}$</td>
<td>10.433</td>
<td>11.870</td>
</tr>
<tr>
<td></td>
<td>$r_{max}$</td>
<td>0.419</td>
<td>0.290</td>
</tr>
<tr>
<td></td>
<td>$P_{max}$</td>
<td>0.999</td>
<td>0.986</td>
</tr>
<tr>
<td>PICRATE results</td>
<td>Mean age</td>
<td>15.42</td>
<td>19.08</td>
</tr>
<tr>
<td></td>
<td>Median age</td>
<td>14.40</td>
<td>17.90</td>
</tr>
<tr>
<td></td>
<td>Variance</td>
<td>7.42</td>
<td>15.40</td>
</tr>
<tr>
<td></td>
<td>Distance from empirical data</td>
<td>0.0098</td>
<td>0.0026</td>
</tr>
<tr>
<td>Distance between both models</td>
<td>0.00012</td>
<td>0.00009</td>
<td>0.00011</td>
</tr>
</tbody>
</table>
7. DISCUSSION

The PICRATE model was developed to fit the age pattern of first marriage in a variety of situations, with the main focus on African countries. The PICRATE model appears as a close approximation to the Coale-McNeil model, offers a simpler formulation of the recruitment rate, and seems to fit empirical data at least as well.

A practical interpretation of the PICRATE model is that the marriage process is determined by an age-specific recruitment rate $r(a)$ from the susceptible population that will eventually marry, that is phased-in from a starting age $a_0$, before which the proportion ever-married and the recruitment rate are zero, to a maximum rate $r_{\text{max}}$ at later ages. Like the Coale-McNeil model, the PICRATE model has only three parameters ($a_0$, $p_{\text{max}}$ and $r_{\text{max}}$), all of which have a simple interpretation: earliest age at marriage, proportion that will eventually marry, and maximum rate of marriage in the susceptible population.

The PICRATE formulation is particularly useful for computer simulations of populations that follow the marriage process through time, since the first marriage recruitment rate $r(a)$ has a simple formula in closed form. Also, the simulated process starts at a definite age $a_0$, with all relevant variable quantities set to zero prior to this point. Furthermore, $r(a)$ tends to $r_{\text{max}}$ as age increases, which suggests that $r_{\text{max}}$ could be a suitable value for the remarriage rate, although this is a matter for further investigation.

Both $p(a)$ and $f(a)$ may be expressed as formulae, almost in closed form, although including the cumulative normal distribution function, which is available in most
mathematical software packages, and for which approximations exist in closed form, such as equation (10) in this article.

The following features of the PICRATE model appear as new and useful. Firstly, the PICRATE recruitment rate has a simple, easily evaluated formula (14), compared with the equivalent Coale-McNeil formula (22). If \( r(a) \) has to be calculated often in a computer program, then the PICRATE formulation has a definite advantage. In an individual-based population computer simulation, it is the probability for each susceptible individual that is necessary to determine transition to first marriage. The marriage rate and the cumulative distribution are needed to verify that the simulation produces acceptable results, but it is the recruitment rate that is needed for the time-evolution.

Secondly, the PICRATE model has zero first marriages before the starting age \( a_0 \), whereas the Coale-McNeil has a finite – though admittedly very small – proportion of first marriages at ages before \( a_0 \), extending to zero age and even to negative ages. More significantly, the Coale-McNeil process requires an initial “seed” starting value of the married proportion, whereas the PICRATE model starts with the proportion at zero. A first marriage model that theoretically allows marriages at all ages, even negative ones, cannot be strictly based on realistic behavioural assumptions. The fact that the Coale-McNeil model does allow such marriages to occur, whereas the PICRATE model does not, suggests that the PICRATE model may have, embedded in its mathematical structure, a more realistic basis than the Coale-McNeil model. This is not surprising, since the Coale-McNeil model was derived as a fit to empirical data, rather than as the result of a behavioural process. The relative simplicity of the PICRATE model formulation suggests that it could turn out to be the result of an underlying theoretical process that is a good model for first marriages.

Our model seems to apply to many similar situations, since its three parameters are very general. Since its development, we have applied it successfully to other processes, such as entry into sexual life (first intercourse), entry into fertile life (first birth), for men as for women. It could also be tried on other similar processes, such as entry into working life (first job), exit from family home (first adult migration), and to many other processes in demography, economy, sociology or epidemiology.

The shape of the Weibull function is simple and asymmetrical, and corresponds to \( k = 2 \), that is the Rayleigh distribution. One could also select other shapes for the recruitment rate, with different parameters of the Weibull function, which imply different asymmetry, to fit other processes such as those which happen at the end of life (end of working life, last migration, etc.).

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